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成蹊大学一般研究報告　第 47 巻第 6 分冊

平成 25 年 11 月

BULLETIN OF SEIKEI UNIVERSITY, Vol. 47 No. 6

November, 2013

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Summary

Universality of the geometrical pomeron is retrospectively sketched at the asymptopia after previous compendia of ours on reggeon-pomeron interaction in proper reference to the hiding cut mechanism *à la* Oehme within the general framework of the reggeized absorption approach based upon the so-called double-partial-wave algorithm in association with the Mandelstam pinch mechanism as well as the Gribov reggeon calculus.

Recent experimental data of the Total and Elastic Measurement (TOTEM) collaboration [1] at the CERN Large Hadron Collider (LHC) as well as cosmic ray measurement by the Pierre Auger Collaboration [2] demonstrates the breakdown of the straightforward extrapolation of the conventional $\ln s$ physics to beyond 10TeV [1-8]. Accordingly the naive phenomenology based upon the idea of the bare simple pole pomeron with the intercept at $t = 0$ slightly above unity is irrelevant at extremely high energies, but instead the concept of the clothed physical pomeron with the unit intercept turns out to be of crucial importance for the self-consistent interpretation of diffractive phenomena at asymptotically high energies [9-12]. The bare pomeron is built up from the normal reggeon through dual topological unitarization. On the other hand, the clothed physical pomeron is generated by multidiffractive unitarization of the bare pomeron. The clothed pomeron is often referred to as the geometrical pomeron (GP) [12]. The GP is universal in the sense that the asymptotic behaviour of the clothed pomeron is independent of the fine details of dynamics building up and unitarizing the bare pomeron. All unusual features of the physical pomeron are commonly inherent in universality of the GP, which plays the role of the most typical guiding principle in pomeron physics. If the GP parametrization is continued in t to beyond the lowest threshold, however, t-channel unitarity is seriously violated because of the hard branching nature. It is then of importance to investigate whether or not the GP universality is self-consistently guaranteed not only from the s-channel point of view but also from the t-channel point of view, and how the universal GP dynamically affects normal reggeons through the repeated pomeron exchange. In the present communication, solutions to these key questions are retrospectively sketched at asymptotically high energies after previous compendia of ours [12] on reggeon-pomeron interaction in proper respect of the so-called hiding cut mechanism (HCM) of Oehme type [13] which might shed some light upon geometrical aspects of a series of our publication on pomeron physics in the 70s. The best possible use is made of the double-partial-wave [DPW] calculus *à la* Bronzan and Sugar [14] which is highly suitable for a perspicacious explanation of geometrical aspects of the reggeized absorption in the presence of the GP. It is taken for granted that any multiple pomeron interaction is correctly described on the basis of the Mandelstam pinch mechanism of the Regge cut generation and the Gribov reggeon calculus of the enhanced Regge cut contribution, *i.e.* the so-called reggeon field theory (RFT) [9-12].

Let us start with the ansatz that the GP amplitude $M_P(s,t)$ is synthetically written in the scaling form

$$\mathrm{Im}M_P(s,t) = 4sy^{-\delta}\,(R(y))^2\,F(\tau) \quad ; \quad \tau = -t\,(R(y))^2 \tag{1}$$

at $s \to \infty$ and $t \to 0$, where $R(y) = r_0 y^\nu$ and $y = \ln(s/s_0)$. The real part $\mathrm{Re}M_P(s,t)$ is obtained from eq. (1) through $s-u$ crossing which reads

$$\mathrm{Re}M_P(s,t) \sim \pi\,(\nu - \delta/2)\,y^{-1}\mathrm{Im}M_P(s,t) \tag{2}$$

in the forward diffraction cone, which is asymptotically negligible at $s \to \infty$ and $t \to 0$. Let us now remember the fundamental constraints on δ and ν. Firstly, unitarity imposes $\delta \geq 0$. Secondly, analyticity requires $0 < \nu \leq 1$. Moreover, the indefinitely rising cross section is realizable if and only if $0 \leq \delta < 2\nu$ which rules out the non-shrinkage of the diffraction peak. This situation is really confirmed by the recent phenomenological observations at LHC energies. Consequently let us postulate $0 \leq \delta < 2\nu$ and $0 < \nu \leq 1$. It is a matter of course that the exact geometrical scaling is assured for the elastic amplitude if and only if $\delta = 0$. The GP partial wave amplitude $f_P(t,J)$ is given by the Mellin transform of eq. (1) which is reduced to be

$$f_P(t,J) = (J-1)^{\delta-2\nu-1}\zeta(\rho) \quad ; \quad \rho = -r_0^2 t(J-1)^{-2\nu}, \tag{3}$$

where

$$\zeta(\rho) = 4s_0 r_0^2 \int_0^\infty dz z^{2\nu-\delta} e^{-z} F(\rho z^{2\nu}). \tag{4}$$

Here, the scaling function ζ satisfies the asymptotic constraints: $\zeta(\rho \to 0) \sim$ constant; $\zeta(\rho \to \infty) \sim \rho^{(\delta-2\nu-1)/2\nu}$. Different GP models disagree with each other only over explicit values of δ and ν, and the interpolating form of ζ. As can easily be seen from eq. (3) as well as the constraints on ζ, every GP trajectory function $\alpha_P(t)$ is uniquely determined by ν through the moving leading singular surface which reads $\rho \sim$ constant, *i.e.*

$$(\alpha_P(t) - 1)^{2\nu} \sim r_0^2 t \quad ; \quad 0 < \nu \leq 1, \tag{5}$$

irrespective of both δ and ζ. The GP partial wave amplitude (3) may then be asymptotically described as

$$f_P(t, J) \sim s_0 r_0^2 \left((J-1)^{2\nu} - r_0^2 t\right)^{(\delta - 2\nu - 1)/2\nu} \quad ; \quad 0 \leq \delta < 2\nu, \quad 0 < \nu \leq 1 \tag{6}$$

in the immediate neighbourhood of $t = 0$, which is literally reduced to the familiar expression of the bare simple pole pomeron in the case of $2\nu = \delta = 1$. Let us postulate $\nu = n/2m$, where $n; m = 1, 2, 3, \cdots$ and $n \leq 2m$. We then obtain

$$\alpha_P^{[j]}(t) = 1 + \left(r_0^2 t\right)^{1/2\nu} \exp\left(\pi i j/\nu\right), \tag{7}$$

where $j = 0, 1, 2, \cdots, n-1$. Since $0 < \nu \leq 1$, $\alpha_P^{[j]}$ is not regular at $t = 0$ except exactly for $\nu = 1/2$, in striking contrast to the bare simple pole pomeron, *i.e.* $2\nu = \delta = 1$. Let us next turn our attention to the t-dependent motion of $\alpha_P^{[j]}$ in the J-plane. Except for integral values of $2\nu - \delta$, all branches $\alpha_P^{[j]}$ coalesce into a hard branch point at $J = 1$ as t tends to 0. If $2\nu - \delta$ is integral, of course, the coalescence at $t = 0$ turns out to be a double pole or a triple pole according as $2\nu - \delta = 1$ or 2. Suppose $t < 0$. Then all pairs of branches with $\mathrm{Re}\alpha_P^{[j]}(t < 0) > 1$ are correctly removed off the physical sheet of the J-plane into unphysical sheets with the aid of the left-hand fixed branch point at $J = 1$ which originates in the factor $(J - 1)^{2\nu}$ with $0 < \nu \neq 1/2 < 1$ of the scaling function ζ. This offers a typical exemplification of the HCM of Oehme type. On the other hand, all branches with $\mathrm{Re}\alpha_P^{[j]}(t < 0) < 1$ are surely allowed to exist in the complex conjugate form on the physical sheet of the J-plane. If $\nu = 1$, ζ has no HCM but instead uniquely yields the self-reproducing, colliding cut pomeron of Schwarz type which reads

$$\alpha_P^{[\pm]}(t) = 1 \pm i r_0 \sqrt{-t} \quad ; \quad i.e. \quad \mathrm{Re}\alpha_P^{[j=0;1]}(t < 0) = 1. \tag{8}$$

Suppose $0 < t < t_0$, where t_0 denotes the t-channel lowest threshold. Then the real positive branch $\alpha_P^{[0]}$, which evidently reads the most right, left-hand branch point, is guaranteed to exist on the physical sheet of the J-plane so long as $r_0 \leq 1/\sqrt{t_0}$. Thus the GP universality is automatically consistent with both s-channel unitarity and the real analyticity as the natural consequence of the HCM of Oehme type. If the GP partial wave amplitude (3) is continued in t to beyond t_0, then t-channel unitarity is inevitably violated because of the hard branching structure at $J = \alpha_P^{[j]}(t)$. The pathological incompatibility of the GP with t-channel unitarity can be remedied, however, by the self-consistent introduction of the shielding cut mechanism (SCM) of Oehme type. We are legitimately led to a model amplitude $f_{[P]}(t, J)$ with the SCM modification

$$f_{[P]}(t,J) = f_P(t,J)\left[\phi_1(t,J) - \frac{\phi_2(t,J)}{\pi}\int_{-\infty}^{\alpha_1(t)} dl \frac{\lambda(t,l) f_P(t,J-l+1)}{(l-1-i\epsilon)\phi_2(t,J-l+1)}\right]^{-1} \tag{9}$$

in the immediate neighbourhood of $t=0$, where

$$\lambda(t,l) = \frac{\alpha_1(t)-l}{[\alpha_1(t)+r_1^2 t_0 - l]^{1/2}}\,, \tag{10}$$

$$\alpha_1(t) = 1 + r_1^2(t-t_0) \tag{11}$$

and $\phi_{1;2}$ are regular, non-vanishing functions. Let us next examine the analytical structure of the model amplitude (9). The $i\epsilon$ procedure guarantees that $f_{[P]}(t,J)$ satisfies t-channel elastic unitarity. The endpoint singularity of the l-integral generates the shielding branch point $\alpha_{SC}^{[j]}(t)$, which reads

$$\alpha_{SC}^{[j]}(t) = 1 + r_1^2(t-t_0) + (r_0^2 t)^{1/2\nu}\exp\left(\pi i j/\nu\right), \tag{12}$$

where $j = 0,1,2,\cdots,n-1$. Since $\phi_{1;2}$ are regular, non-vanishing, every shielding branch point at $J = \alpha_{SC}^{[j]}(t)$ makes a soft contribution to $f_{[P]}(t,J)$, irrespective of the fine details of the branching character. Thus all shielding branch points $\alpha_{SC}^{[j]}$ are fully consistent with t-channel unitarity. As J tends to $\alpha_P^{[j]}$, then the moving branch point $t_{SC} = \{\alpha_{SC}^{[j]}\}^{-1}(J)$ exactly coincides with t_0. Consequently the cut running from t_{SC} completely shields the t-channel elastic branch cut in the limit of $J \to \alpha_P^{[j]}$. With the aid of the shielding machinery of the branches $\alpha_{SC}^{[j]}$, all pairs of hard branch points $\alpha_P^{[j]}$ are removed from the physical sheet of the J-plane under the continuation of $f_{[P]}(t,J)$ into the second sheet of the t-plane. Thus the model amplitude (9) successfully satisfies the continuity theorem. It is therefore possible to conclude that the shielding branch point $\alpha_{SC}^{[j]}$ correctly satisfies the principal machinery of the SCM of Oehme type and that the GP ansatz is always made compatible with t-channel elastic unitarity and the continuity theorem by the best possible use of the SCM. It is of importance to note that phenomenological consequences of the soft branch point $\alpha_{SC}^{[j]}$ are legitimately negligible at the asymptopia compared with those of the hard GP branch point $\alpha_P^{[j]}$. Accordingly the self-similarity of the GP amplitude (1), which is the most important realization of universality of the clothed physical pomeron, is asymptotically not destroyed by the self-consistent introduction of the SCM of Oehme type.

The impact parameter profile function $a_P(s,b)$ of the GP is given by the Fourier-Bessel transform of eq. (1) which yields

$$\mathrm{Im}a_P(s,b) = y^{-\delta}\varphi\left(b/R(y)\right), \tag{13}$$

where

$$\varphi(\xi) = \int_0^\infty dz z J_0(\xi z) F(z^2) \quad ; \quad \xi = b/r_0 \cdot y^{-\nu}. \tag{14}$$

The total and elastic cross sections read

$$\sigma_{tot}(s) = 8\pi r_0^2 y^{2\nu-\delta} \int_0^\infty d\xi \xi \varphi(\xi) \tag{15}$$

and

$$\sigma_{el}(s) = 8\pi r_0^2 y^{2\nu-2\delta} \int_0^\infty d\xi \xi \left(\varphi(\xi)\right)^2, \tag{16}$$

respectively. Equation (13) literally describes the scaled shape of the GP opacity distribution. Accordingly the scaling parameter ξ in the b-plane uniquely corresponds to the scaling variable ρ in the t-plane which is crucially responsible for the structure of the singular surface (5). Thus the HCM of the GP is commonly inherent in the geometrical scaling. Let us remember the fundamental constraints on the scaling function φ, *i.e.* (i) the unitarity bound $0 < \varphi(0) \leq 1/2$ in the case $\delta = 0$, (ii) the analyticity bound $\varphi(\xi) \leq O\left(\exp(-\xi)\right)$ at sufficiently large ξ and (iii) the duality requirement of the non-peripheral distribution of $\varphi(\xi)$. Thus the impact parameter profile of the GP is correctly described by a smooth- or sharp-edged disc whose radius increases sufficiently fast and whose central shape of opacity is sufficiently flat. It is parenthetically mentioned that the magnitude of opacity is saturated at $b = 0$ with the unitarity upper bound for the limiting case of both $\delta = 0$ and $\varphi(0) = 1/2$, *i.e.* the so-called perfect absorption, in which low b-waves are completely absorbed at very high energies, *i.e.* $\text{Im}a(s, b = 0) \sim 1/2$ at $s \to \infty$, or equivalently

$$\int_0^\infty d\tau F(\tau) = 1 \quad ; \quad s \to \infty \tag{17}$$

which in turn reads

$$\int_0^\infty d\tau \text{Im} M_P(s,t) = 4s \left(R(y)\right)^2 \tag{18}$$

at asymptotically high energies. The sum rule (17) or equivalently (18) has been confirmed *à la* Srivastava [6] by the TOTEM data. We then eventually set up $\delta = 0$. If and only if $\delta = 0$, the ratio $\sigma_{el}(s)/\sigma_{tot}(s)$ is asymptotically s-independent as follows:

$$\sigma_{el}(s)/\sigma_{tot}(s) = \int_0^\infty d\xi \xi \left(\varphi(\xi)\right)^2 / \int_0^\infty d\xi \xi \varphi(\xi) \tag{19}$$

in full agreement with the argument of Maor [15] at $\sqrt{s} \simeq 1.8 \sim 100$TeV. If and only if $\delta = 0$, in addition, the ratio $\sigma_{tot}(s)/8\pi\langle b^2\rangle$ is asymptotically s-independent as follows:

$$\sigma_{tot}(s)/8\pi\langle b^2\rangle = \left(\int_0^\infty d\xi \xi \varphi(\xi)\right)^2 / \int_0^\infty d\xi \xi^3 \varphi(\xi), \tag{20}$$

where $\langle b^2\rangle$ reads the mean square radius of the opacity distribution in terms of which the slope parameter $B(s)$ of the forward elastic peak is expressed as

$$B(s) = \frac{d}{dt} \ln\left(d\sigma_{el}/dt\right)|_{t=0} = \frac{1}{2}\langle b^2\rangle$$
$$= \frac{1}{2} r_0^2 y^{2\nu} \int_0^\infty d\xi \xi^3 \varphi(\xi) / \int_0^\infty d\xi \xi \varphi(\xi). \tag{21}$$

Thus the exact geometrical scaling, *i.e.* $\delta = 0$, is inevitable for the scaling behaviour of the opaqueness of the hadronic-matter distribution undergoing a high-energy collision. The self-similarity of the shape of the hadronic-matter distribution is guaranteed, irrespective of δ and ν, in the general case of the GP, however. Unitarity requires both $0 \leq \sigma_{el}(s)/\sigma_{tot}(s) \leq 1/2$ and $0 \leq \sigma_{tot}(s)/8\pi\langle b^2\rangle \leq 1/2$ which are satisfied under the ansatz of a sufficiently bounded ξ-distribution of the scaling function φ. The experimental values of σ_{el}/σ_{tot} and $\sigma_{tot}/8\pi\langle b^2\rangle$ are much smaller than the unitarity upper bound 1/2, *i.e.* the so-called black disc limit, even at the LHC energies. It is parenthetically mentioned that the unitarity upper bound, *i.e.* the so-called black disc limit reads

$$\varphi(\xi) = \frac{1}{2}\theta(1-\xi) \quad ; \quad \xi = b/r_0 \cdot y^{-1} \tag{22}$$

in addition to $\delta = 0$ and $\nu = 1$ in accordance with the sharp cut-off, hadronic matter distribution, which yields

$$\mathrm{Im} M_P(s,t) = 2 s r_0^2 y^2 \frac{J_1\left(r_0 y \sqrt{-t}\right)}{r_0 y \sqrt{-t}} \tag{23}$$

and

$$f_P(t,J) = \frac{2 s_0 r_0^2}{\left[(J-1)^2 - r_0^2 t\right]^{3/2}} . \tag{24}$$

We then obtain

$$\sigma_{tot}(s) = 2\pi r_0^2 y^2 \tag{25}$$

and

$$\sigma_{el}(s) = \pi r_0^2 y^2 . \tag{26}$$

Thus saturation of the celebrated Froissart upper bound on the total cross section is substantiated as an inevitable consequence of the self-reproducing, colliding cut pomeron $\alpha_P^{[\pm]}(t)$ of Schwarz type (8). Implications of saturation of the Froissart bound at the LHC energies and beyond were elaborated by Block [8].

Let us now sketch diffractive dissociation in full accordance with the GP universality. At least from the phenomenological point of view, inelastic collision consists of diffractive (D) and non-diffractive (ND) components. The ND component dominates over the D component in multi-particle production. The ND component is in turn dominantly controlled through short range rapidity correlation (SRRC) mechanism with the minor correction from long range rapidity correlation (LRRC) mechanism. The bare ND overlap function is described through the SRRC component and reasonably well represented by the factorizable, simple pole pomeron with the intercept $\alpha_P(0)$ of which is slightly above unity. From the theoretical point of view, therefore, absorptive correction are inevitable in order to guarantee the celebrated Froissart bound at asymptotically high energies. The LRRC component is then obtained as a result of the second step absorptive unitarization of the divergent SRRC component. Moreover the D component is generated as a natural consequence of the shadow effect of the ND component within the general framework of the absorptive unitarization. Theoretical features of the solution of the absorptive unitarization are epitomized from the point of view of the GP universality. The D states are labelled $i; j; k = 1, 2, 3, \cdots$. In particular, the elastic state is designated as $i; j; k = 1$. Impact parameter profiles of the D amplitude and ND overlap function between i and j states are designated as $H_{ij}(s, b)$ and $M_{ij}(s, b)$, respectively. The absorptive s-channel unitarity is then written in the form

$$2H_{ij}(s,b) = \sum_k H_{ik}(s,b)H_{kj}(s,b) + \sum_k \left(\delta_{ik} - H_{ik}(s,b)\right) M_{kj}(s,b), \tag{27}$$

under the ansatz of reality of both $H_{ij}(s, b)$ and $M_{ij}(s, b)$ at the asymptopia. Let us suppose the s-channel factorizability of $M_{ij}(s, b)$ in the sense of

$$M_{ij}(s,b) = \gamma_i(s,b)\gamma_j(s,b). \tag{28}$$

The matrix $\mathbf{M}(s, b)$ then has one and the only one non-vanishing eigenvalue which reads

$$\lambda(s,b) = \sum_i \left(\gamma_i(s,b)\right)^2 . \tag{29}$$

The matrix $\mathbf{H}(s, b)$ is simultaneously diagonalizable in terms of the complete set of eigenstates of $\mathbf{M}(s, b)$ and yields

$$\begin{aligned} H_{ij}(s,b) &= h(s,b)/\lambda(s,b) \cdot \gamma_i(s,b)\gamma_j(s,b) \\ &= h_i(s,b)h_j(s,b), \end{aligned} \tag{30}$$

where the unique non-vanishing eigenvalue $h(s, b)$ reads

$$h(s,b) = \left(2 + \lambda(s,b) - \left(4 + (\lambda(s,b))^2\right)^{1/2}\right)/2 . \tag{31}$$

Thus the D amplitude between D states is correctly obtained as the shadow effect of totality of ND transitions between D and all possible ND states. The D component is described in terms of the factorizable ND overlap function. The SRRC dominance in multi-hadronic production and $\alpha_P(0) > 1$ for the bare pomeron yield inevitably the divergent, central b-distribution of the ND overlap function at the asymptopia. As a consequence, the divergence of the central distribution of $M_{11}(s,b) = (\gamma_1(s,b))^2$; $0 < \xi \lesssim 1$, is supposed in natural correspondence to the SRRC dominance in multiparticle production and $\alpha_P(0) > 1$ for the bare pomeron. Since $\lambda(s,b) \geq (\gamma_1(s,b))^2$, eq. (31) immediately yields

$$h(s,b) \simeq 1 - (\lambda(s,b))^{-1} \quad ; \quad 0 < \xi \lesssim 1 \tag{32}$$

at $s \to \infty$. The central distribution of the impact parameter profiles $\sigma_{tot}(s,b)$, $\sigma_{el}(s,b)$, $\sigma_{inel;D}(s,b)$ and $\sigma_{inel;ND}(s,b)$ then turn out to be

$$\sigma_{tot}(s,b) = 2\,(\gamma_1(s,b))^2 / \lambda(s,b), \tag{33}$$

$$\sigma_{el}(s,b) = (\gamma_1(s,b))^4 / (\lambda(s,b))^2, \tag{34}$$

$$\sigma_{inel;D}(s,b) = (\gamma_1(s,b))^2 / \lambda(s,b) \times \left(1 - (\gamma_1(s,b))^2 / \lambda(s,b)\right) \tag{35}$$

and

$$\sigma_{inel;ND}(s,b) = (\gamma_1(s,b))^2 / \lambda(s,b), \tag{36}$$

respectively, at the asymptopia. Equations (33), (34), (35) and (36) bring forth

$$\sigma_{el}(s,b) + \sigma_{inel;D}(s,b) = \sigma_{inel;ND}(s,b) = 1/2 \cdot \sigma_{tot}(s,b) \quad ; \quad 0 < \xi \lesssim 1, \tag{37}$$

or equivalently

$$(\sigma_{el}(s) + \sigma_{inel;D}(s)) / \sigma_{tot}(s) = \sigma_{inel;ND}(s)/\sigma_{tot}(s) = 1/2 \tag{38}$$

at asymptotically high energies. Accordingly saturation of the so-called Pumplin bound on the D component is materialized as the unique solution of the absorptive s-channel unitarity. The inelastic D component is generated and stabilized in association with the elastic component through the shadow effect of the inelastic ND component. Thus there seems to be no persuasive reason to claim that the GP contribution is significantly different between $\sigma_{el}(s)$ and $\sigma_{inel;D}(s)$. It is otherwise impossible to make a well-defined distinction between the D and the ND mechanisms within the general framework of the absorptive unitarization. It is of interest to note that the asymptotic relation (38) is qualitatively not too far from the experimental information at the LHC energies, *i.e.* $\sigma_{el} \sim$ 25mb, $\sigma_{inel;D} \sim$ 15mb and $\sigma_{tot} \sim$ 100mb [4]. The concept of universality plays the role of the most important guiding principle in pomeron physics. We postulate by universality that the asymptotic behaviour of the GP is independent of the fine details of the promotion mechanism of the bare pomeron. Let us parenthetically remind once again the diffractive single channel approximation in which $\lambda(s,b) = (\gamma_1(s,b))^2$; $0 < \xi \lesssim 1$. Then the sharp-edged, complete black disc pomeron provides the unique solution of eq. (27), which offers a naive exemplification of universality as well as the maximization of $\sigma_{tot}(s)$ and inevitably yields

$$\sigma_{el}(s) = \sigma_{inel;ND}(s) = 1/2 \cdot \sigma_{tot}(s) = \pi r_0^2 y^2 \tag{39}$$

at asymptotically high energies. Saturation of eq. (39) is too far from the experimental information even at the LHC energies. Consequently the diffractive single channel approximation may be of no interest at least from the phenomenological point of view. Thus we are naturally led to the diffractive many channel paradigm. Then the ratio $(\gamma_1(s,b))^2/\lambda(s,b)$ in turn cannot be uniquely determined just through the general properties of the absorptive s-channel unitarity. Accordingly the absorptive unitarization is not so sufficiently powerful as to guarantee automatically the GP universality in the diffractive many channel algorithm. Fundamental physics underlying the GP universality undoubtedly deserves more than passing consideration.

We are now confronted with an interesting problem: how the GP universality affects normal reggeons in reggeon-pomeron interaction. Both the Mandelstam pinch mechanism and the Gribov reggeon calculus provide us with the standard machinery which yields a typical materialization of the GP universality in reggeon-pomeron dynamics [9-12]. The leading corrections to any normal reggeon through the repeated pomeron exchange are in fact estimated as the effect of the simultaneous exchange of the GP and the normal reggeon. In order to clarify the principal machinery of the universal GP in the GP-reggeon dynamics, let us remember the discussion on the GP parametrization (3). The t-dependence of the singular surface (5) is uniquely determined through the scaling parameter ρ. The GP is then described as just one moving leading singular surface, irrespective of the detailed branching structure. Therefore the forward scattering amplitude of the GP exchange is asymptotically factorizable in the standard manner as the consequence of the scaling form (3). From the aesthetic point of view, let us assume that the input amplitude $M_R(s,t)$ of the normal reggeon exchange is synthetically written in the scaling form

$$\mathrm{Im} M_R(s,t) = \gamma s_0 \left(s/s_0\right)^{\alpha} F_R(\tau_R) \quad ; \quad \tau_R = -\alpha' t y^{2\nu_R} \tag{40}$$

at $s \to \infty$ and $t \to 0$, where $0 < \nu_R = \delta_R/2 \le 1$. The partial wave amplitude $f_R(t,J)$ of the normal reggeon exchange is synthetically expressed by

$$f_R(t,J) = (J-\alpha)^{-1} \zeta_R(\rho_R) \quad ; \quad \rho_R = -\alpha' t \, (J-\alpha)^{-2\nu_R} \, , \tag{41}$$

where

$$\zeta_R(\rho_R) = \gamma s_0 \int_0^{\infty} dz e^{-z} F\left(\rho_R z^{2\nu_R}\right) . \tag{42}$$

Here, the scaling function ζ_R satisfies the asymptotic constraints: $\zeta_R\left(\rho_R \to 0\right) \sim$ constant; $\zeta_R\left(\rho_R \to \infty\right) \sim \rho_R^{-1/2\nu_R}$. The reggeon trajectory function $\alpha_R(t)$ then satisfies the moving leading singular surface which reads $\rho_R \sim$ constant, *i.e.*

$$(\alpha_R(t)-\alpha)^{2\nu_R} \sim \alpha' t \quad ; \quad 0<\nu_R\le 1. \tag{43}$$

That is, the normal reggeon is controlled by just one moving leading singular surface, irrespective of the fine details of the branching nature. Accordingly factorizability of the input forward scattering amplitude of the normal reggeon exchange is asymptotically guaranteed in the usual sense as the consequence of the scaled form (41). The impact parameter profile function $a_R(s,b)$ of the normal reggeon exchange is expressed as

$$\mathrm{Im}a_R(s,b)=\gamma/4\alpha'\cdot(s/s_0)^{\alpha-1}y^{-2\nu_R}\varphi_R(\xi_R) \quad ; \quad \xi_R=b/\sqrt{\alpha'}\cdot y^{-\nu_R} \tag{44}$$

in accordance with the scaled shape of the opacity distribution of the normal reggeon, where

$$\varphi_R(\xi_R)=\int_0^\infty dzzJ_0(\xi z)F_R(z^2). \tag{45}$$

Our purpose is reduced to the examination of the structure of the clothed leading singular surface in the output partial wave amplitude $f_{RP}(t,J)$ which originates from the simultaneous exchange of the GP and the normal reggeon. As a valid generalization, hereafter, the parameter δ is tentatively considered as a free parameter, not fixed at $\delta=0$, in the present context. It is almost needless to mention that the simultaneous exchange of α_R and α_P is successfully described at the asymptopia in terms of the modified profile function

$$\mathrm{Im}\tilde{a}_R(s,b)\approx(s/s_0)^{\alpha-1}y^{-\delta-2\nu_R}\tilde{\varphi}_R(\xi_R;\xi) \quad ; \quad \tilde{\varphi}_R(\xi_R;\xi)\sim\varphi_R(\xi_R)\varphi(\xi) \tag{46}$$

at sufficiently high energies, where the double exchange mechanism of Mandelstam type has been postulated for $0\le\delta<2\nu$, $0<\nu\le 1$ and $0<\nu_R=\delta_R/2\le 1$, in general. Since

$$\xi_R=\xi\cdot r_0/\sqrt{\alpha'}\cdot y^{\nu-\nu_R}, \tag{47}$$

the ratio ξ/ξ_R eventually tends to 0 or ∞ in the limiting case of $s\to\infty$ according to whether $\nu>\nu_R$ or $\nu<\nu_R$. In consequence, the two antipodal cases: (i) $0<\nu_R<\nu\le 1$ and (ii) $0<\nu<\nu_R\le 1$ can be examined in the completely symmetric manner. Let us suppose the case (i) [(ii)]. We then obtain $\tilde{\varphi}_R\sim\varphi_R$ [$\tilde{\varphi}_R\sim\varphi$] at the asymptopia. Therefore the leading singular surface of the Mellin-Fourier-Bessel transform $f_{RP}(t,J)$ of the profile function (46) is asymptotically controlled just by the scaling parameter ξ_R [ξ] or equivalently by the scaling variable $\rho_R=-\alpha' t(J-\alpha)^{-2\nu_R}$ [$\tilde{\rho}_R=-r_0^2 t(J-\alpha)^{-2\nu}$]. The output, leading reggeon trajectory function $\tilde{\alpha}_R(t)$ arising from the simultaneous exchange of α_R and α_P satisfies the moving leading singular surface

$$\begin{cases} (\tilde{\alpha}_R(t)-\alpha)^{2\nu_R}\sim\alpha' t & ; \quad 0<\nu_R<\nu\le 1 \\ (\tilde{\alpha}_R(t)-\alpha)^{2\nu}\sim r_0^2 t & ; \quad 0<\nu<\nu_R\le 1, \end{cases} \tag{48}$$

irrespective of δ, ζ and ζ_R. The forward amplitude of the output reggeon exchange is then factorizable in the conventional fashion, irrespective of the detailed branching structure of the output, moving leading singular surface (48). The principal conclusion in the case (i) [(ii)] is as follows. If the branching nature of the trajectory function of the input reggeon is less [more] singular at $t = 0$ than that of the GP, then the output reggeon carries universally the same trajectory function as that of the input reggeon [then the output reggeon carries the trajectory function, the t-dependence [the intercept] of which reads universally the same as that of the GP [the input reggeon]]. In order to obtain a deeper understanding of the consistency of these results with the celebrated Mandelstam pinch mechanism, let us consider the special case $0 < \nu_R = \nu < 1$. The standard Mandelstam mechanism is straightforwardly applicable to this example and yields the output, leading singular surface

$$(\tilde{\alpha}_R(t) - \alpha)^{2\nu} \sim \tilde{\alpha}' t \quad ; \quad 0 < \nu_R = \nu < 1, \tag{49}$$

where

$$\tilde{\alpha}' = \alpha' \left(1 + (\alpha'/r_0^2)^{1/2(1-\nu)}\right)^{2(\nu-1)} \tag{50}$$

which is reduced to the familiar expression

$$\tilde{\alpha}' = \alpha' r_0^2 / \left(\alpha' + r_0^2\right) \tag{51}$$

in the limiting case of $\nu_R = \nu = 1/2$. Factorization of the output forward amplitude is guaranteed in the ordinary manner. Equation (43) is formally written in the form

$$(\alpha_R(t) - \alpha)^{2\nu} \sim \bar{\alpha}' t, \tag{52}$$

where

$$\bar{\alpha}' = (\alpha')^{\nu/\nu_R} t^{(\nu-\nu_R)/\nu_R}. \tag{53}$$

Since we are primarily interested in the immediate neighbourhood of $t = 0$, the case (i) [(ii)] legitimately corresponds to the special example mentioned above in the limit $\bar{\alpha}'/r_0^2 \to 0$ [$\bar{\alpha}'/r_0^2 \to \infty$]. If α' is replaced by $\bar{\alpha}'$ in eq. (50), then $\tilde{\alpha}' \to \bar{\alpha}'$ or $\tilde{\alpha}' \to r_0^2$ according to whether $\bar{\alpha}'/r_0^2 \to 0$ or $\to \infty$. Thus the surface (48) is correctly identifiable with the limiting case of eq. (49) and evidently obeys the Mandelstam generating mechanism of Regge cuts. Accordingly the aforementioned, apparently antipodal phenomena are not only fully compatible with each other but also furnish the typical substantiation of universality of the GP in pomeron-reggeon interaction. Elaboration of the Regge cut generation is requisite for the case $\nu = 1$ and/or $\nu_R = 1$, however. For the detailed discussion, we merely refer to ref. [12; Riv.].

In order to clarify the fundamental aspects of the absorptive mechanism of the GP, let us assume the most standard parametrization of the single-reggeon exchange amplitude $M_R(s,t)$:

$$\mathrm{Im} M_R(s,t) = \gamma s_0 \exp\left[\alpha_R(t) y\right], \tag{54}$$

i.e. $\nu_R = \delta_R/2 = 1/2$, where

$$\alpha_R(t) = \alpha + \alpha' t. \tag{55}$$

We then obtain immediately the partial wave amplitude $f_R(t,J)$ and the impact parameter profile function $a_R(s,b)$ as follows:

$$f_R(t,J) = \frac{\gamma s_0}{J - \alpha_R(t)} \tag{56}$$

and

$$\mathrm{Im} a_R(s,b) = \frac{\gamma}{8\alpha' y} \exp\left[(\alpha-1)y - b^2/4\alpha' \cdot y^{-1}\right], \tag{57}$$

respectively. The best possible use is made of the DPW algorithm in which the DPW amplitude $a(J,b)$ is defined by the Fourier-Bessel transform of the partial wave amplitude $f(t,J)$:

$$a(J,b) = \frac{1}{4s_0}\int_0^\infty d\sqrt{-t}\sqrt{-t} J_0\left(b\sqrt{-t}\right) f(t,J), \tag{58}$$

or equivalently by the Mellin transform of the impact parameter profile function $a(s,b)$:

$$a(J,b) = \int_0^\infty dy \exp\left[-(J-1)y\right] \mathrm{Im} a(s,b). \tag{59}$$

Accordingly the DPW amplitude $a_P(J,b)$ of the GP is reduced to be

$$\begin{aligned} a_P(J,b) = \frac{1}{\nu}\left(\frac{b}{r_0}\right)^{(1-\delta)/\nu} & \int_0^\infty d\xi \xi^{(\delta-\nu-1)/\nu} \varphi(\xi) \\ \times \exp\left[-(J-1)(b/r_0 \cdot \xi^{-1})^{1/\nu}\right] & \quad ; \quad 0 \le \delta/2 < \nu \le 1. \end{aligned} \tag{60}$$

Similarly the DPW amplitude $a_R(J,b)$ turns out to be

$$a_R(J,b) = \frac{\gamma}{4\alpha'} K_0\left(b\sqrt{(J-\alpha)/\alpha'}\right), \tag{61}$$

where K_0 is the modified Bessel function of order zero. The absorbed, partial wave amplitude $f_{[R]}(t,J)$ can be written in the form

$$f_{[R]}(t,J) = f_R(t,J) + f_{RP}(t,J) \tag{62}$$

within the framework of the reggeized absorption approach, where $f_{RP}(t,J)$ means the partial wave amplitude in association with the simultaneous exchange of the GP and the normal reggeon. By making use of the DPW amplitudes $a_P(J,b)$ and $a_R(J,b)$, $f_{RP}(t,J)$ is expressed as

$$f_{RP}(t,J) = 8s_0 \iint_{c-i\infty}^{c+i\infty} \frac{dl_1 dl_2}{(2\pi i)^2} \frac{1}{J+1-l_1-l_2}$$
$$\times \int_0^\infty dbbJ_0\left(b\sqrt{-t}\right) a_P(l_1,b) a_R(l_2,b) \quad ; \quad 0 \le \delta/2 < \nu \le 1, \tag{63}$$

where the signature factor of the reggeon has been left out of consideration in the present context and the l_1 $[l_2]$ contour is chosen so as to enclose, in the clockwise [counter-clockwise] direction, the pole at $J = l_1 + l_2 - 1$ [the branch cut of K_0 running along the real axis from $-\infty$ to α]. The partial wave amplitude $f_{RP}(t,J)$ is then reduced to be

$$f_{RP}(t,J) = -\frac{\gamma s_0}{\alpha'\nu} r_0^{(\delta-1)/\nu} \int_0^\infty d\xi \xi^{(\delta-\nu-1)/\nu} \varphi(\xi)$$
$$\times \int_0^\infty dbb^{-(\delta-\nu-1)/\nu} J_0\left(b\sqrt{-t}\right) \int_0^\infty dx J_0\left(b\sqrt{x/\alpha'}\right)$$
$$\times \exp\left[-(J-\alpha+x)(b/r_0 \cdot \xi^{-1})^{1/\nu}\right], \tag{64}$$

where use has been made of the relation [14]

$$K_0(iz) - K_0(-iz) = -i\pi J_0(z) \tag{65}$$

in the l_2 integration of eq. (63). The inverse Mellin transform of eq. (64) yields the absorptive reggeon amplitude $M_{RP}(s,t)$ as follows:

$$\mathrm{Im}M_{RP}(s,t) = -\frac{\gamma s_0 r_0^2}{\alpha'} y^{2\nu-\delta-1} e^{\alpha y} \int_0^\infty d\xi \xi \varphi(\xi) J_0\left(\xi r_0 y^\nu \sqrt{-t}\right)$$
$$\times \exp\left[-\frac{\xi^2 r_0^2}{4\alpha'} y^{2\nu-1}\right] \quad ; \quad 0 \le \delta/2 < \nu \le 1. \tag{66}$$

The absorbed reggeonic amplitude $M_{[R]}(s,t)$ is then literally written in the form

$$\mathrm{Im}M_{[R]}(s,t) = \mathrm{Im}M_R(s,t) + \mathrm{Im}M_{RP}(s,t). \tag{67}$$

Thus our purpose is reduced to the detailed estimation of eq. (66) at asymptotically high energies under the ansatz of an explicit ξ-dependence of φ. We take it for granted that φ correctly satisfies the aforementioned fundamental restrictions, *i.e.* (i) $0 < \varphi(0) \le 1/2$ in the case $\delta = 0$, (ii) $\varphi(\xi) \le O\left(e^{-\xi}\right)$ at sufficiently large ξ and (iii) the non-peripheral distribution of $\varphi(\xi)$. The intercept at $t = 0$ of the trajectory function of the absorptive leading reggeon is then uniquely given by α, irrespective of δ, ν and φ, which is consistent with the fact that absorptive Regge singularities are generated through the Mandelstam mechanism. Unless the leading singular surface of eq. (48) carries about the input trajectory function $\alpha_R(t)$, therefore, the physical region in which absorptive Regge singularities are dominated by the input reggeon is confined to an s-dependent, immediate neighbourhood of $-t \simeq 0$ which exactly shrinks to $t = 0$ at $s \to \infty$. In the conventional case of the bare simple pole pomeron, *i.e.* $2\nu = \delta = 1$, indeed, the domain mentioned above asymptotically reads

$$-t \lesssim \left(\alpha' + r_0^2\right)/\alpha'^2 \cdot \ln\ln(s/s_0)/\ln(s/s_0). \tag{68}$$

However, our primary concern is the possible predominance of the input reggeon over absorptive leading Regge singularities at least in a s-independent, finite physical domain. Consequently our next step is to elaborate the possible criterion for discriminating whether or not the trajectory function $\tilde{\alpha}_R(t)$ of the absorptive leading Regge singularity is surely described by $\alpha_R(t)$.

Explicit estimation of the absorptive reggeonic amplitude (66) is performed at the asymptopia in the presence of a wide class of smooth- or sharp-edged disc GP. Let us first examine the reggeized absorption in the presence of the smooth-edged disc GP with the shape function $\varphi(\xi)$ which reads

$$\varphi(\xi) = \varphi(0)\exp\left(-\xi^{2\mu}\right) \quad ; \quad \mu \geq 1/2, \tag{69}$$

where $0 < \varphi(0) \leq 1/2$ in the case $\delta = 0$. Suppose $\mu > 1$. Irrespective of δ, then, the asymptotic evaluation eq. (66) yields the absorptive leading Regge singularity with $\alpha_R(t)$ if and only if $1/2 \leq \nu \leq 1$. The result reads

$$\begin{aligned}\mathrm{Im}M_{RP}(s,t) \simeq& -2\gamma s_0\varphi(0)y^{-\delta}\\ &\times\left\{e^{\alpha_R(t)y} - \frac{\mu-1}{\mu}e^{\alpha y}\exp\left[-\left(\frac{r_0^2}{4\alpha'}y^{2\nu-1}\right)^{\mu/(\mu-1)}\right]\right.\\ &\left.\times J_0\left(\left(\frac{r_0^2}{4\alpha'}y^{2\mu\nu-1}\right)^{1/2(\mu-1)}\left(-r_0^2t\right)^{1/2}\right)\right\} \quad ; \quad 1/2 \leq \nu \leq 1\end{aligned} \tag{70}$$

at extremely high energies. Suppose $1/2 \leq \mu \leq 1$. Irrespective of δ, then, we obtain asymptotically the absorptive leading Regge trajectory function $\alpha_R(t)$ if and only if $1/2 < \nu \leq 1$. Equation (66) results in

$$\begin{aligned}\mathrm{Im}M_{RP}(s,t) \simeq& -2\gamma s_0\varphi(0)y^{-\delta}\\ &\times\left\{e^{\alpha_R(t)y} - \frac{1}{2}\frac{1-\mu}{1+\mu}e^{\alpha y}\left(\frac{r_0^2}{4\alpha'}y^{2\nu-1}\right)^{2\mu/(\mu-1)}\right\} \quad ; \quad 1/2 < \nu \leq 1\end{aligned} \tag{71}$$

at asymptotically high energies. In the other cases of both $\mu > 1$; $0 < \nu < 1/2$ and $1/2 \leq \mu \leq 1$; $0 < \nu \leq 1/2$, however, the absorptive leading Regge trajectory function $\tilde{\alpha}_R(t)$ satisfies

$$\left(\tilde{\alpha}_R(t) - \alpha\right)^{2\nu} \propto r_0^2 t \quad ; \quad 0 < \nu \leq \nu_R = 1/2 \tag{72}$$

in association with the criterion (ii). In both of these cases, therefore, the asymptotic behaviour of the absorbed reggeonic amplitude $M_{[R]}(s,t)$ is dominantly described, irrespective of δ, by a set of absorptive Regge singularities, instead of the input reggeon. It is parenthetically mentioned that the hypothetical ansatz of the scaled, dipole structure of the hadronic matter distribution *à la* Chou and Yang yields

$$\varphi(\xi) = \varphi(0)/8 \cdot \xi^3 K_3(\xi) \quad ; \quad \xi = b/r_0 \cdot y^{-\nu}, \tag{73}$$

where K_3 reads the modified Bessel function of order 3. The tail contribution of $\varphi(\xi)$ then turns out to be

$$\varphi(\xi) \sim \xi^{5/2} e^{-\xi} \tag{74}$$

in sharp contrast to the Gaussian distribution as well as the sharp cut-off distribution. Let us next investigate the reggeized absorption in the presence of the sharp-edged disc GP with the shape function $\varphi(\xi)$ which reads

$$\varphi(\xi) = \begin{cases} \varphi(0) \exp\left(-\xi^{2\mu}\right) \theta\left(\xi_0 - \xi\right) & ; \quad \mu \geq 1/2 \\ \varphi(0) \theta\left(\xi_0 - \xi\right) & ; \quad \mu = 0, \end{cases} \tag{75}$$

where $0 < \varphi(0) \leq 1/2$ in the case $\delta = 0$. Suppose $\mu > 1$. Irrespective of δ, then, the absorptive leading Regge singularity is accompanied with $\alpha_R(t)$ if and only if $1/2 \leq \nu \leq 1$. Equation (66) brings forth

$$\begin{aligned} \mathrm{Im} M_{RP}(s,t) \simeq & -2\gamma s_0 \varphi(0) y^{-\delta} \\ & \times \left\{ e^{\alpha_R(t) y} - N\left(s,t,\xi_0,\nu\right) \right\} \quad ; \quad 1/2 < \nu \leq 1 \end{aligned} \tag{76}$$

and

$$\begin{aligned} \mathrm{Im} M_{RP}(s,t) \simeq & -2\gamma s_0 \varphi(0) y^{-\delta} \\ & \times \left\{ e^{\alpha_R(t) y} - \theta\left(\xi_0 - \left(\frac{r_0^2}{4\alpha'}\right)^{1/2(\mu-1)}\right) \left[\frac{\mu-1}{\mu} N\left(s,t,\left(\frac{r_0^2}{4\alpha'}\right)^{1/2(\mu-1)}, \nu = \frac{1}{2}\right) \right. \right. \\ & \left. + \frac{r_0^2}{4\alpha'\mu} \xi_0^{2-2\mu} \exp\left[\frac{\xi_0^2 r_0^2}{4\alpha'} - \xi_0^{2\mu}\right] N\left(s,t,\xi_0,\nu=\frac{1}{2}\right) \right] \\ & \left. - \theta\left(\left(\frac{r_0^2}{4\alpha'}\right)^{1/2(\mu-1)} - \xi_0\right) N\left(s,t,\xi_0,\nu=\frac{1}{2}\right) \right\} \quad ; \quad \nu = 1/2 \end{aligned} \tag{77}$$

in replacement of eq. (70), at sufficiently high energies, where

$$N\left(s,t,\xi_0,\nu\right) = e^{\alpha y} \exp\left(-\frac{\xi_0^2 r_0^2}{4\alpha'} y^{2\nu-1}\right) J_0\left(\xi_0 y^{\nu} \left(-r_0^2 t\right)^{1/2}\right). \tag{78}$$

Suppose $1/2 \leq \mu \leq 1$. Irrespective of δ, then, the absorptive leading Regge singularity carries about $\alpha_R(t)$ if and only if $1/2 < \nu \leq 1$. Equation (66) gives rise to

$$\begin{aligned} \mathrm{Im} M_{RP}(s,t) \simeq & -2\gamma s_0 \varphi(0) y^{-\delta} \\ & \times \left\{ e^{\alpha_R(t) y} - \frac{1}{2}\frac{1-\mu}{1+\mu} e^{\alpha y} \left(\frac{r_0^2}{4\alpha'} y^{2\nu-1}\right)^{2\mu/(\mu-1)} \right. \\ & \left. - N\left(s,t,\xi_0,\nu\right) \right\} \quad ; \quad 1/2 < \nu \leq 1, \end{aligned} \tag{79}$$

instead of eq. (71), at very high energies. Suppose $\mu = 0$. Irrespective of δ, then, the absorptive leading Regge trajectory function $\alpha_R(t)$ is obtained if and only if $1/2 \leq \nu \leq 1$. The asymptotic estimation of eq. (66) yields

$$\mathrm{Im}M_{RP}(s,t) \simeq -2\gamma s_0\varphi(0)y^{-\delta} \times \left\{e^{\alpha_R(t)y} - N(s,t,\xi_0,\nu)\right\} \quad ; \quad 1/2 \leq \nu \leq 1 \tag{80}$$

at extremely high energies. So far the signature factor of the reggeon has bean thoroughly kept out of consideration. Let us suppose the standard ghost-killing mechanism in accordance with the analytical structure of $a_P(J,b)$. Then the criterion for ascertaining the possible predominance of the input simple pole reggeon over absorptive leading Regge singularities is irrespective of the detailed structure of the signature of the reggeon. On the basis of these observations, we are led to the following salient conclusions with regard to the reggeized absorption in the presence of a wide class of GP. Firstly, irrespective of the s-dependence of the magnitude of opacity, the absorptive leading Regge singularity is accompanied with the input trajectory function $\alpha_R(t)$ if and only if the radius of the GP disc expands not less fast than $(\ln(s/s_0))^{1/2}$. The input reggeon is then completely cancelled in the absorbed, partial wave amplitude $f_{[R]}(t,J)$ by the absorptive leading Regge singularity of $\alpha_R(t)$ if and only if the radius of the GP disc expands not less fast than $(\ln(s/s_0))^{1/2}$ and the magnitude of opacity is saturated at $b=0$ with the unitarity upper bound, *i.e.* $\delta=0$ and $\varphi(0)=1/2$. In the case of the GP disc whose radius grows faster then $(\ln(s/s_0))^{1/2}$, both of these situations are attainable, irrespective of the ξ-dependence of the central shape of opacity. In the case of the GP disc whose radius grows like $(\ln(s/s_0))^{1/2}$, however, both of these situations are realizable if and only if the ξ-dependence of the shape of opacity is flatter than the Gaussian distribution at least in the region $b \lesssim r_0(\ln(s/s_0))^{1/2}$. Secondly, irrespective of the s-dependence of the magnitude of opacity, the asymptotic behaviour of the absorbed reggeonic amplitude $M_{[R]}(s,t)$ is dominantly controlled by the input reggeon, at least in a s-independent, finite physical domain which reads $-t \lesssim r_0^2/4\alpha'^2$ for the smooth-edged disc GP and $-t \lesssim \xi_0^2 r_0^2/4\alpha'^2$ for the sharp-edged disc GP, respectively, if and only if the radius of the GP disc stretches faster than $(\ln(s/s_0))^{1/2}$, the magnitude of opacity is not completely black even at $b=0$ and the ξ-dependence of the central shape of opacity is not less flat than the Gaussian distribution. In the case of the smooth-edged disc GP whose central shape of opacity is described by the Gaussian distribution, this is attainable if and only if the radius extends faster than $(\ln(s/s_0))^{1/2}$. In the case of the smooth-edged disc GP whose central shape of opacity is more black than the Gaussian distribution, this is attainable if and only if the radius extends more and more fast according as the central shape becomes flatter. Strictly speaking, $(2\mu-1)/2\mu < \nu \leq 1$ in the case $\mu > 1$. Consequently ν tends to 1 as μ unboundedly increases. It is parenthetically mentioned that the flat sharp-edged disc GP can legitimately be identified with the limiting case of $\mu=\infty$, *i.e.* $\nu=\xi_0=1$. In particular, in the case of the smooth-edged disc GP whose radius extends like $\ln(s/s_0)$, this is realizable if and only if the shape of opacity is not less flat than the Gaussian distribution in the region $b \lesssim r_0\ln(s/s_0)$. Similarly, in the case of the sharp-edged disc GP, this is achievable if and only if the radius increases like $\ln(s/s_0)$ and the shape of opacity is not less flat than the Gaussian distribution in the region $b \lesssim r_0 \ln(s/s_0)$. Thirdly, irrespective of the s-dependence of the magnitude of opacity, the asymptotic behaviour of the absorbed reggeonic amplitude $M_{[R]}(s,t)$ is dominated by a set of absorptive Regge singularities in the whole physical region of the t-plane except for a s-dependent, immediate neighbourhood of $-t \simeq 0$ which exactly shrinks to $t = 0$ at $s \to \infty$, unless the radius of the GP disc increases not less fast than $(\ln(s/s_0))^{1/2}$ and the central shape of opacity is not less flat than the Gaussian distribution. Let us recapitulate, by way of parenthesis, that the asymptotic behaviour of the absorbed reggeonic amplitude is controlled by the input simple pole reggeon, irrespective of the s-dependence of the magnitude of opacity of the GP disc, at least in a s-independent, physical domain of the t-plane, if and only if the impact parameter profile of the GP is described by a smooth- or sharp-edged disc whose radius stretches sufficiently fast, whose magnitude of opacity is not completely black even at $b = 0$ and whose central shape of opacity shows sufficiently flat distribution.

Let us summarize major features of the GP. Firstly, the HCM of Oehme type naturally originates in the scaling form of the GP amplitude and correctly guarantees the consistency of the GP universality with s-channel unitarity, the real analyticity and the asymptotic factorizability. Secondly, the self-similarity of the GP is made fully compatible with t-channel unitarity by the self-consistent introduction of the SCM of Oehme type. Thirdly, the singular surface of the output reggeon remains the same as that of the input reggeon or simulates that of the GP with the exception of the intercept at $t = 0$ under the GP-reggeon interaction, according to whether the trajectory function of the input reggeon is less singular at $t = 0$ than that of the GP or more singular at $t = 0$. Let us turn our attention to major aspects of the reggeized absorption in the presence of the GP. The recent phenomenological observation at the LHC energies and beyond suggest that the total cross section $\sigma_{tot}(s)$ asymptotically rises faster than $\ln(s/s_0)$ in addition to the so-called perfect absorption. We then postulate $1/2 < \nu \leq 1$, $\delta = 0$ and $\varphi(0) = 1/2$ without loss of generality. On the basis of the present discussion on reggeon-pomeron interaction, therefore, it will be possible to predict that the absorptive leading Regge singularity carries the trajectory function $\alpha_R(t)$ of the input simple pole reggeon, that the input simple pole reggeon is completely cancelled by the absorptive leading reggeon of $\alpha_R(t)$ in the absorbed, partial wave amplitude $f_{[R]}(t, J)$ and that the absorbed reggeonic amplitude $M_{[R]}(s,t)$ is asymptotically described by a set of absorptive Regge singularities instead of the input reggeon. High-statistics experimental data on hadronic forward scattering are requisite at the next LHC energies, *e.g.* 14TeV for examining these predictions in association with the postulate mentioned above. Phenomenology based upon the pomeron has been very successful in describing high-energy soft interaction during half a century. We are now in a position to build up a unified model of the pomeron which not only describes high-energy soft interaction at low k_T in terms of the so-called soft pomeron, *i.e.* the vacuum exchange object, but also applies to the high k_T pQCD domain as a smooth transition into the so-called hard pomeron or equivalently the pQCD pomeron, *i.e.* the sum of ladder diagrams of interacting reggeized gluons [16]. The partonic structure of the bare pQCD pomeron is extrapolated into the soft region and reproduces the main features of the soft pomeron as an inevitable consequence of significant absorptive corrections in the sense of the multi-pomeron effect. Various intriguing models are now available, *e.g.* conventional parametrization analysis [5], QCD mini-jet paradigm [6], QCD inspired RFT algorithm [7], QCD inspired multi-channel eikonal model [16], QCD colour-glass-condensate picture [17] and AdS/CFT approach [18]. In particular, the pomeron in $N = 4$ SUSY may be dual-symmetrically described *à la* Lipatov [18] as the reggeized graviton on the AdS space. Thus the pomeron calculus might be reduced to the algorism based upon the effective action for the ensemble of reggeized gravitons. It is undoubtedly of supreme interest in the pomeron geometrodynamics to clarify whether or not gravity sheds some new light upon fundamental physics underlying the GP universality.

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Dr. M. Shimano of Jumonji Gakuen is highly thanked for typing up the present manuscript. Professor T. Kon is acknowledged for the cordial hospitality at Seikei University.

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